TEST NAME: **G-GMD.1**
TEST ID: **464130**
GRADE: **09**
SUBJECT: **Mathematics**
TEST CATEGORY: **My Classroom**

Student:

Class:

Date:

1. **Pi Day**

In this task, you will use measurements and areas of polygons to derive the formulas for the circumference and area of a circle.

You can use a ruler to measure distances that are straight, but it is difficult to measure around a circle with a ruler. However, you can do an experiment to find a good approximation of the formula for the circumference of a circle.

You can find the area of a rectangle by calculating the number of square units that cover it, but square units do not fit neatly in a circle. However, you can approximate the area of a circle by finding the area of known figures that are close in size.

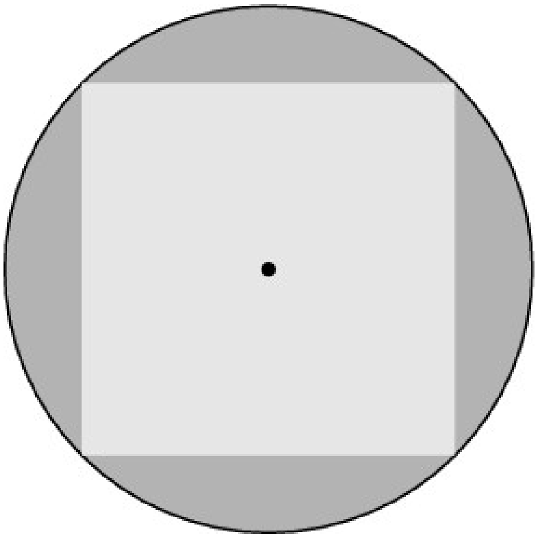
Part A. Use the circular or cylindrical items your teacher provides. Wrap a string around each item and measure the length of string that goes around it. Record this measure as the circumference of the item and include units. Then, use a ruler or yardstick to measure the diameter of the circle. Record your findings in the table below and include units. You may use different units for different items, but be sure that both measurements of each item are in the same units. Be as precise as possible when you are measuring.

Item	Circumference (C)	Diameter (d)	Ratio (C/d)
Average of Ratios			

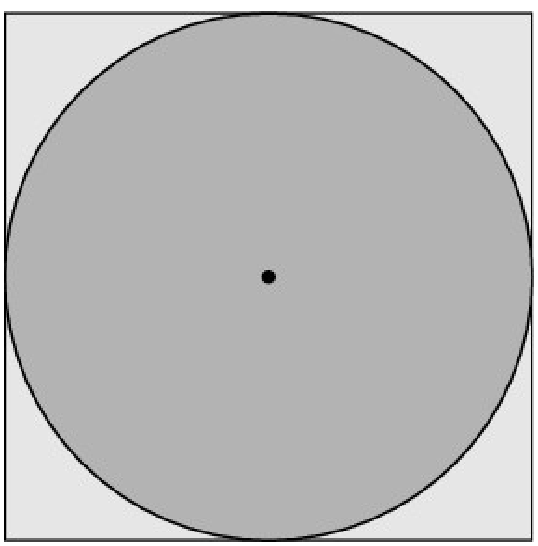
Part B. In the last column of the table, write the ratio of the circumference to the diameter for each item. In the last row, write the average of all 10 ratios. Use the information in your table to write a formula to find the circumference, C, of a circle if you know its diameter, d. Explain your answer. How does this relate to the formula for a circumference of a circle that you normally use? Explain any differences.

Now, we will find a way to calculate the area of a circle.

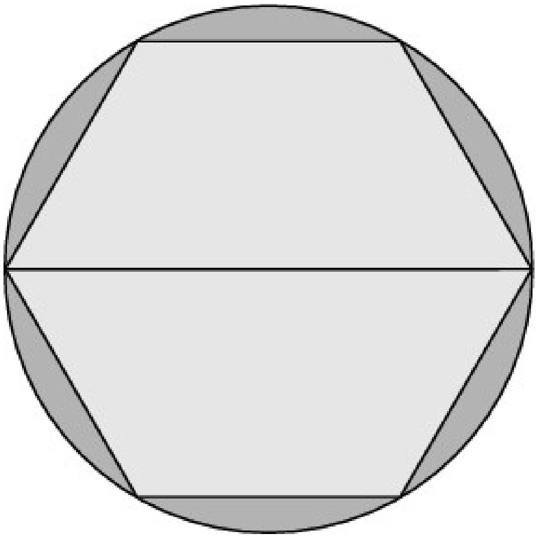
Part C. If *r* is the radius of the circle below, what is the area of the square inscribed in it? Show your work and write your answer in terms of *r*.



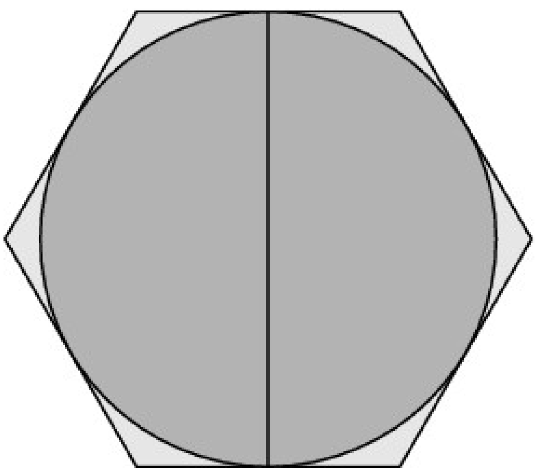
Part D. A square that is circumscribed around a circle has an area that is greater than the area of the circle. Find the area of this square in terms of the radius of the circle, *r*. Use your conclusions from parts C and D to write an inequality that compares the area of the circle, A_c, with the area of this square and the area of the square in part C.



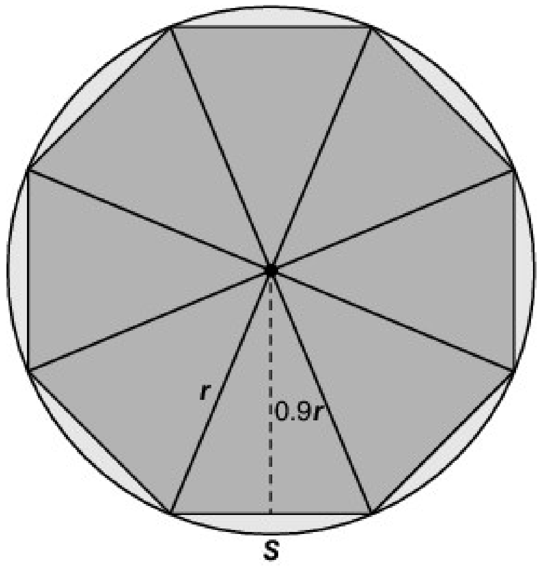
Part E. A regular hexagon inscribed in the circle would have an area closer to that of the circle. Find the area of the hexagon in terms of the radius of the circle, *r.* Round your answer to the nearest tenth and explain your work.



Part F. Find the area of a hexagon that is circumscribed around a circle. Write your answer in terms of the radius, *r*. Round your answer to the nearest tenth and explain your work. Then, use your conclusions from parts E and F to write an inequality for the area of the circle, A_c, based on the areas you found in parts E and F.

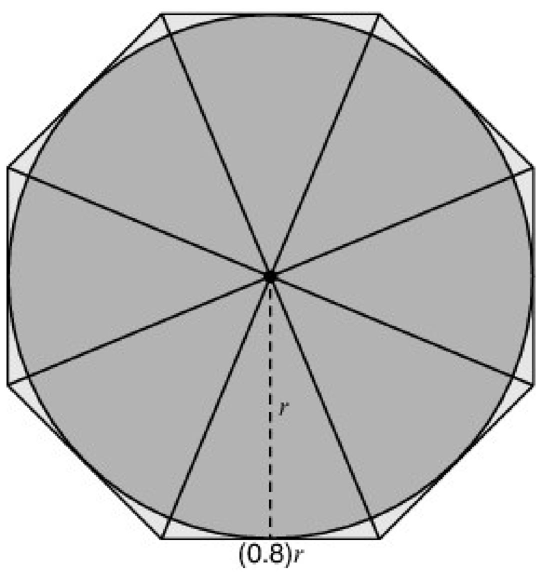


Part G. An inscribed regular octagon is even closer to the area of the circle. The height of one of the little triangles is about $0.9r$. What is the approximate area of the octagon in terms of *r*? Round your answer to the nearest tenth and explain your work.



Part H. For a circumscribed octagon, the base of one of the little triangles is about $0.8r$. What is the approximate area of the octagon in terms of *r*?

Rund your answer to the nearest tenth. Based on what you found in parts G and H, write a new inequality for the area of the circle.



Part I. For a polygon that is inscribed or circumscribed in a circle, the greater the number of sides, the closer the area of the polygon is to the area of the circle. Suppose the polygon has *n* sides, each with a length of *s*. The polygon can be decomposed into *n* little triangles. Let the height of each little triangle be *h*. Write a formula to give the area of the polygon in terms of *n*, *s*, and *h*.

$A =$ ____

As *n* gets bigger and bigger, what measure of the circle does the height of the little triangles approach?

What measure of the circle does $n \times s$ approach?

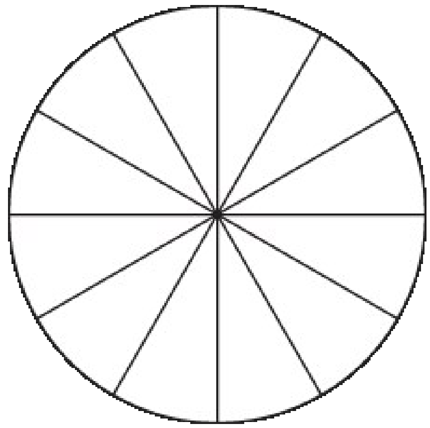
Rewrite your formula for the area of the circle by replacing *h* and $n \times s$ with these values.

$A =$ ____

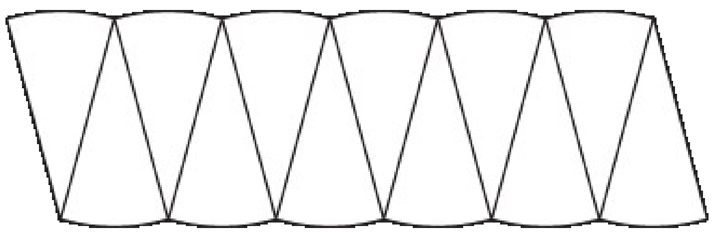
Show how to simplify the formula to give the standard form for the area of a circle. Think about how to express the circumference (*C*) in terms of the radius (*r*).

2. The students in a geometry class use the area of a parallelogram to informally determine the formula for the area of a circle. Their steps are as shown below.

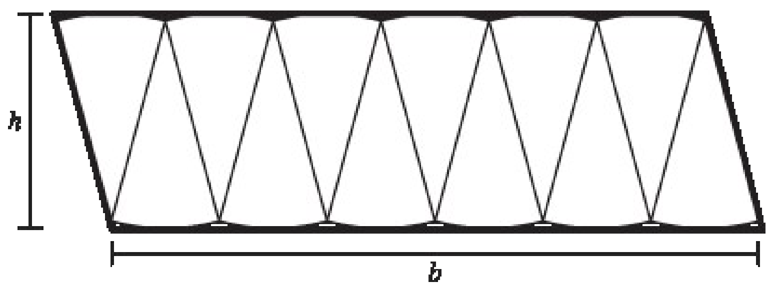
Step 1: Divide a circle into 12 wedges of equal size.



Step 2: Rearrange the wedges alternately up and down to resemble the shape of a parallelogram.



Step 3: Determine the dimensions of the parallelogram.



The height, h, of the parallelogram approximates the radius, r, of the circle.

What part of the circle is approximated by the base of the parallelogram?

A. one-sixth of the circumference

B. one-half of the circumference

C. the circumference

D. twice the circumference

3. Rachel is finding the formula for the area of a circle. She folded a circular paper plate 4 times to get 16 equal-sized sections.

- Sketch a circle and show the 16 equal-sized sections Rachel created with her plate.
- Rachel cut out all of the sections and rearranged them to form a shape that looks almost like a parallelogram. Sketch her new arrangement.
- She noticed that the radius of a circle represents the height of the shape. Which measure of the circle describes the length of the base of the parallelogram?
- Using the formula for the area of a parallelogram and the measures from above, derive the formula for the area of a circle. Show your steps.

Use words, numbers, and/or pictures to show your work.

4. A food company currently sells tomato juice in a cylindrical container with a diameter of 6 centimeters (cm) and a height of 10 cm. The company's marketing team suggests there would be better sales if the container were a cone. What should the diameter of the cone be if the company changes neither the height nor the volume? Round to the nearest cm, if needed.

A. 5

B. 9

C. 11

D. 27

5. Which best describes how to find the volume of a pyramid?

A. multiply the area of the base by the height

B. multiply the area of the base by the height and by $\frac{1}{2}$

C. multiply the area of the base by the height and by $\frac{1}{3}$

D. multiply the area of the base by the height and by the slant height

6. If a rectangular pyramid has the same height and base as a rectangular prism, which statement explains how the volume of the pyramid can be determined?

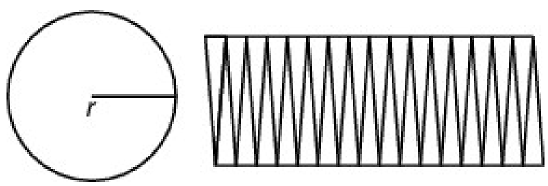
A. The volume of the pyramid is one-half the volume of the prism.

B. The volume of the pyramid is one-third the volume of the prism.

C. The volume of the pyramid is one-fourth the volume of the prism.

D. The volume of the pyramid is the same as the volume of the prism.

7. The circle with radius *r* units is divided into congruent sectors to create the shape shown below.



For estimation purposes, the shape formed by the sectors is considered to be a parallelogram. Which statement is **true**?

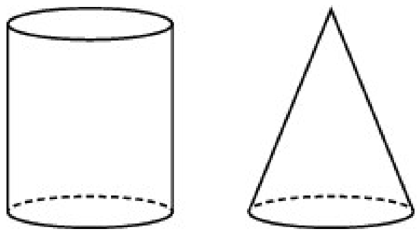
A. The height is approximately *r* units, and the base length is approximately πr units.

B. The height is approximately *r* units, and the base length is approximately 2π units.

C. The height is approximately r^2 units, and the base length is approximately π units.

D. The height is approximately $2r$ units, and the base length is approximately π units.

8. The cylinder and the cone shown below have the same height, and their bases have the same radius.



How does the volume of the cylinder (V_{cyl}) compare to the volume of the cone (V_{cone})?

A. $(V_{cyl}) = 2V_{cone}$

B. $V_{cyl} = \frac{1}{3}V_{cone}$

C. $V_{cyl} - V_{cone} = 3V_{cone}$

D. $V_{cyl} - V_{cone} = 2V_{cone}$

9. A certain cylinder has a height of h_c, and a certain square prism has a height of h_p. The circular cross section of the cylinder and the square cross section of the prism have the same area. Which equation expresses the relationship of the volume of the cylinder, V_c, to the volume of the prism, V_p?

A. $V_c = V_p$

B. $\pi V_c = V_p$

C. $\frac{V_c}{V_p} = \frac{h_p}{h_c}$

D. $\frac{V_c}{V_p} = \frac{h_c}{h_p}$

10. Jenny is given the task of carving out a triangular pyramid and a square pyramid of the same height from two identical wooden blocks. She first needs to determine the dimensions of each pyramid before starting to carve out the blocks.

Part 1
She decides that the triangular pyramid will have a right triangle as its base with legs of lengths a and $2a$, and its height will be h. Use Cavalieri's principle to determine the formula for the volume of the triangular pyramid.

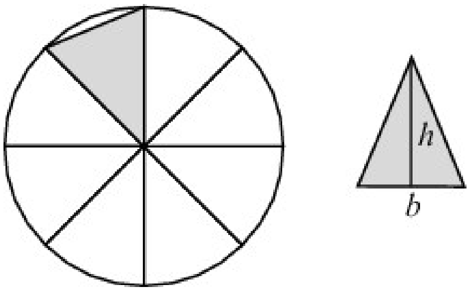
Part 2
She decides to carve out a square pyramid with a base of side $2b$. Use Cavalieri's principle to determine the formula for the volume of the square pyramid.

Part 3
What is the relation between a and b?

Part 4
If Jenny finally decides to carve out the triangular pyramid such that its longer leg measures 12 inches (in.) and its height is 25 in., what will be the area of the base of the square pyramid? What will be the volume of the square pyramid?

Use words, numbers, and/or pictures to show your work.

11. A circle of radius *r* is divided into *n* equal sections. In the figure below, $n = 8$. The area of the circle can be estimated by thinking of each pie-shaped piece as a triangle with base *b* and height *h*.



The more sections (*n*) the circle is divided into, the closer the approximation of the area. As *n* gets larger, which statements must be true?

A. *h* approaches *r*
bn approaches πr

B. *h* approaches *r*
bn approaches 2πr

C. *n* approaches *r*
bh approaches πr

D. *n* approaches *r*
bh approaches 2πr

12. Margaret is using a hollow cone and a hollow cylinder to determine how the volume of a cone relates to the volume of a cylinder. The heights and diameters of the bases of the cone and the cylinder are equal. Margaret fills the cone completely with water and then pours the water into the cylinder. What will she find?

A. The water fills $\frac{1}{3}$ of the cylinder.

B. The water fills $\frac{1}{2}$ of the cylinder.

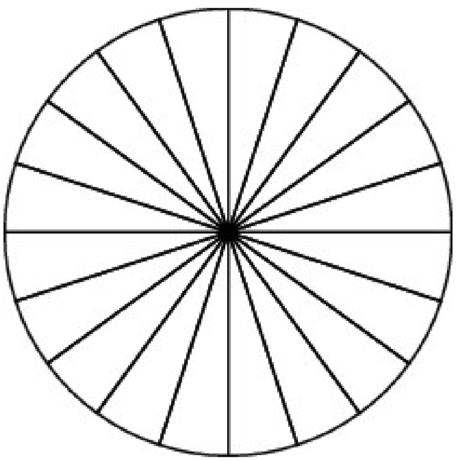
C. The water will fill the cylinder to the top.

D. The water fills $\frac{2}{3}$ of the cylinder.

13. An interior decorator is repurposing a circular wooden tabletop and turning it into a piece of art. The circular tabletop has a diameter of 2 feet, and the decorator plans to use all of the tabletop in his piece of art.

Part A. What is the area of the circular tabletop? What is the circumference? Show your work and round your answers to the nearest hundredth of a foot.

Part B. The decorator cuts along the lines shown below so that the the circular tabletop is divided into 20 congruent sectors.



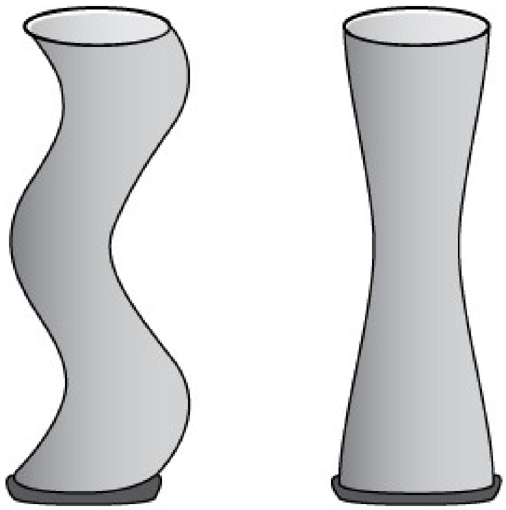
After treating the wooden sectors so that he can paint them, he places the sectors next to each other, alternating the direction of each piece so that the resulting shape is almost a parallelogram. Sketch what the parallelogram will look like, using dashed lines to show the border of each sector and solid marks to show the border of the piece of art.

Part C. What are the approximate dimensions of the parallelogram-shaped piece of art? Explain. How does the area of the piece of art compare with the area of the original circle? Explain. Show your work and round your answers to the nearest hundredth.

Part D. When the decorator treated the wooden pieces, he put them in a bucket that contained exactly 1 cubic foot of chemical solution. Once the pieces were immersed, the water level rose and the bucket contained 1.2 cubic feet. To the nearest hundredth of a cubic foot, what is the volume of wood the tabletop was composed of? Given the volume of the wooden pieces, what was the depth of the tabletop? Show your work and round your answer to the nearest hundredth of a foot.

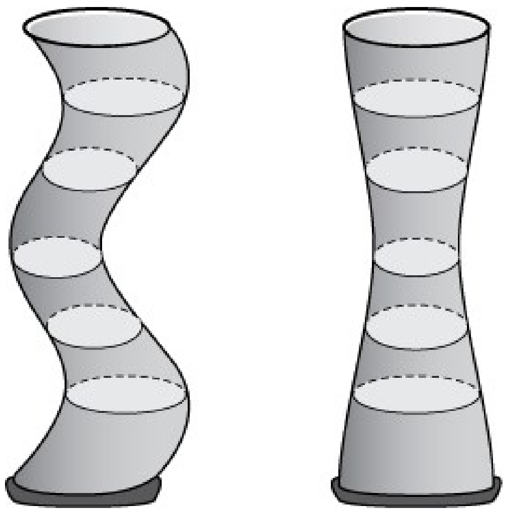
Use words, numbers, and/or pictures to show your work.

14. Two vases have the same height but are shaped differently as shown below.

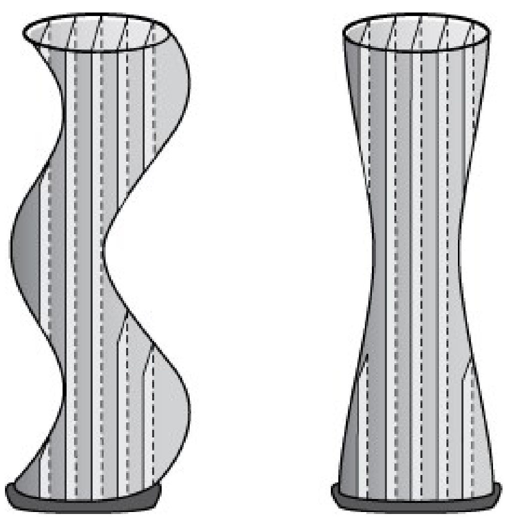


How could the vases be sliced to use Cavalieri’s principle to show that their volumes are equivalent?

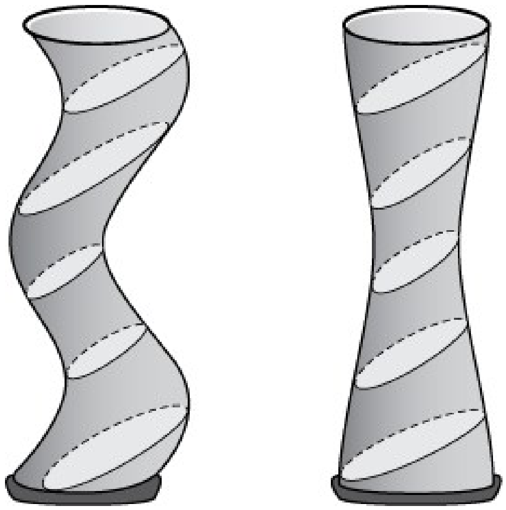
A.



B.



C.



D.