Math I Cheat Sheet 2017:

Distance Formula

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint Formula

$(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2})$

Slope Formula:

$\frac{rise}{run}$ $\frac{y_2-y_1}{x_2-x_1}$

Slope-Intercept Form:

$y = mx + b$

where m = slope and b = y-intercept

Types of Slope:

Positive Negative Zero (Horizontal Line) Undefined (Vertical Line)

Quadratic Equation:

$ax^2 + bx + c$

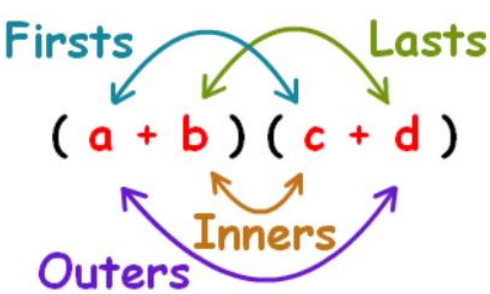
Axis of Symmetry of Quadratic:

$\frac{-b}{2a}$

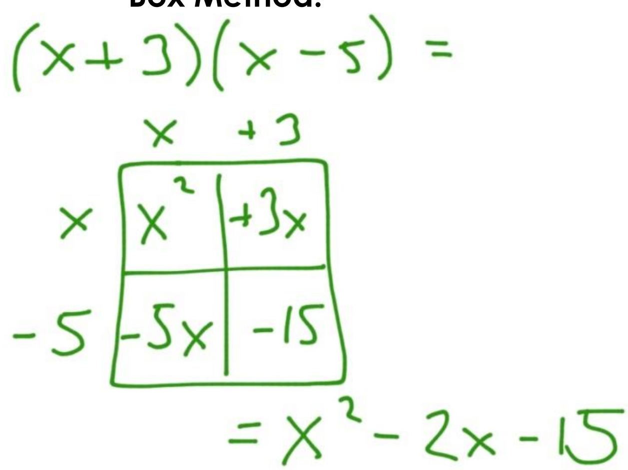
Polynomials

Multiplying Polynomials:

Foil:



Box Method:



Special cases

$(a+b)^2$	$(a-b)^2$	$(a+b)(a-b)$
$(a+b)(a+b)$	$(a-b)(a-b)$	$a^2 - ab + ab - b^2$
$a^2 + 2ab + b^2$	$a^2 - 2ab + b^2$	$a^2 - b^2$

Degree-The exponent on a term tells you the "degree" of the term. The degree is the highest power of an exponent in a polynomial.

Factoring:

Basic Trinomial: $ax^2 + bx + c$ *where a is* 1.

1) identify a,b, and c in the trinomial $ax^2 + bx+c$

2) write down all factor pairs of c

3) identify which factor pair from the previous step sums up to b

4) Substitute factor pairs into two binomials

Factoring Trinomials $ax^2 + bx + c$ *where a is* > 1. [Factor by Grouping]

<u>Steps:</u>

Step 1: Make sure that the trinomial is written in the correct order; the trinomial must be written in descending order from highest power to lowest power.

Step 2 : Decide if the three terms have anything in common, called the greatest common factor or GCF. If so, factor out the GCF. Do not forget to include the GCF as part of your final answer.

Step 3 : Multiply the leading coefficient and the constant, that is multiply the first and last numbers together.

Step 4 : List all of the factors from Step 3 and decide which combination of numbers will combine to get the number next to x.

Step 5 : After choosing the correct pair of numbers, you must give each number a sign so that when they are combined they will equal the number next to x and also multiply to equal the number found in Step 3.

Step 6 : Rewrite the original problem with four terms by splitting the middle term into the two numbers chosen in step 5.

Step 7 : Now that the problem is written with four terms, you can factor by grouping.

$$\underbrace{2wx + 10w}_{\text{factor}} + \underbrace{7x + 35}_{\text{factor}}$$

$$= 2w(x+5) + 7(x+5)$$

$$= (x+5)(2w+7) \quad \text{done!}$$

Laws of Exponents

Multiplying Powers of the Same Base:

If you are **multiplying** powers of the **same base**, you just **add the exponents**.

$$(x^a)(x^b) = x^{a+b}$$

$$(xxx)(xxxxx) = x^8$$

or

$$(x^3)(x^5) = x^{3+5} = x^8$$

Raising a Power to a Power:

Any power of a power: you multiply the exponents.

$$(x^a)^b = x^{ab}$$

$$(x^2)^4 = x^{(2)(4)} = x^8$$

Or

$$(x^2)^4 = (x^2)(x^2)(x^2)(x^2) = (xx)(xx)(xx)(xx) = x^8$$

Zero Power of Exponent:

Anything to the 0 power is 1.

$$x^0 = 1$$

Dividing Powers of the Same Base:

Division with like bases you subtract exponents.

$$\frac{x^a}{x^b} = x^{a-b}$$

For example, $\frac{5^5}{5^3} = 5^{5-3} = 5^2 = 25$

$$\frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5} = 5 \cdot 5 = 25$$

Negative Exponents:

A negative exponent means to **divide** by that number of factors **instead of multiplying**.

So 4^{-3} is the same as $\frac{1}{4^3}$, and $x^{-3} = \frac{1}{x^3}$.

As you know, **you can't divide by zero**. So there's a restriction that $x^{-n} = \frac{1}{x^n}$ only when x is not zero. When x = 0, x^{-n} is undefined.

Fractional Exponents:

A fractional exponent—an exponent of the form $\frac{1}{n}$ —means to **take the nth root.**

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

For example, $4^{\frac{1}{3}}$ is the 3rd root (cube root) of 4 = $\sqrt[3]{4}$.

Radicals:

Splitting up a Square Root:

$$(ab)^{\frac{1}{2}} = \sqrt{ab} = (\sqrt{a})(\sqrt{b})$$

Simplifying a Square Root:

$$\sqrt{ab} = (\sqrt{a})(\sqrt{b})$$

For example:

$$\sqrt{32} = (\sqrt{16})(\sqrt{2}) = 4\sqrt{2}$$

Simplifying Multiplying with Square Roots:

$$(\sqrt{a})(\sqrt{b}) = \sqrt{ab}$$

For example:

$$\sqrt{12}\sqrt{8} = \sqrt{96}$$

Simplifying Division with Square Roots:

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

For example:

$$\sqrt{\frac{16}{9}} = \frac{\sqrt{16}}{\sqrt{9}} = \frac{4}{3}$$

Addition and Subtract with Square Roots:

$$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$$

And

$$\sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$$

Summary & Guidelines to "Jail Break:"

1) Create a factor tree for the number in jail.
2) Keep factoring until you have ONLY PRIME numbers- circle them.
3) You can only break out if you have a pair of prime numbers- it must be exactly the same numbers.
4) Only 1 person in the pair actually survives the jail break. The other person dies, but does not go back to jail.
5) Anybody who doesn't find a partner to break out of jail, must stay in! Multiply any leftover numbers together and keep them in jail!

Example:

$$\sqrt{48x^3y^6} = \sqrt{6 \cdot 8 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y}$$
$$= \sqrt{3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y}$$
$$= 2 \cdot 2 \cdot x \cdot y \cdot y \cdot y\sqrt{3x}$$
$$= 4xy^3\sqrt{3x}$$

Pythagorean Theorem: $a^2 + b^2 = c^2$

Exponential Function

$y = a(b^x)$ where a is the y-intercept and b is the growth/decay factor
-If b is greater than 1 then it is exponential growth
-If b is less than 1 then it is exponential decay

Growth : 1 + r　　　　Decay: 1- r

Growth/Decay Rate- the percent of increase/decrease
Growth/Decay Factor- what you multiply by

Domain and Range

Domain- all the x-values　　　　Range- all the y-values

Linear Graphs- $y = mx + b$
Domain- all real numbers
Range- all real numbers

Exponential Graphs- $y = a(b^x)$
Domain- all real numbers
Range- all positive values

Quadratic Graphs- $ax^2 + bx + c$
Domain- all real numbers
Range- +,- infinity to min/max value

Square Root Graphs- $y = \sqrt{x}$
Domain- all positive numbers
Range- all positive numbers

Systems of Equations

One Solution- two linear lines that cross only once

Infinite Solutions- two lines that are exactly the same

No Solution- Two parallel lines (lines never cross)

Elimination Method:

$$\begin{Bmatrix} x + y = 2 \\ x - y = 14 \end{Bmatrix}$$

$$\begin{array}{r} x + y = 2 \\ x - y = 14 \\ \hline 2x \quad = 16 \end{array}$$

← eliminate the y variable by adding equations

$2x = 16$ ← solve for x

$x = 8$ ← use to find y

$$x + y = 2$$

$$\begin{array}{r} 8 + y = 2 \\ -8 \quad\quad -8 \\ \hline y = -6 \end{array}$$

(8, −6) solution

$$\begin{cases} -x + 5y = 8 \\ 3x + 7y = -2 \end{cases} \overset{\times 3}{\Rightarrow} \begin{cases} -3x + 15y = 24 \\ 3x + 7y = -2 \end{cases}$$

$$\begin{array}{r} -3x + 15y = 24 \\ + \quad 3x + 7y = -2 \\ \hline 22y = 22 \end{array}$$

$$\frac{22y}{22} = \frac{22}{22}$$

$$y = 1$$

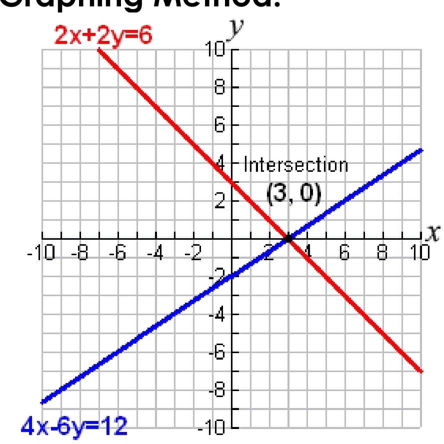
Substitution Method:

Directions: Solve the following system of equations using substitution.

-x +y = 1
2x +y = -2

Step 1: -x +y = 1 -x +x +y = 1 +x y = 1 +x y = x +1	Solve 1 equation for 1 variable: (x = …) or (y=…) I chose the first equation because it was the easiest to rewrite. I added x to each side to rewrite this equation as y = x+1.
Step 2: 2x +y = -2 ↘ 2x + x+1 = -2 3x + 1 = -2 3x +1 -1 = -2 -1 $\frac{3x}{3} = \frac{-3}{3}$ x = -1	Substitute this expression into the other equation and solve. Since I know that y = x+1, I substituted x+1 for y into the equation, 2x +y = -2. Then I solved for x and found x = -1
Step 3: y = x +1 y = -1 +1 y = 0	Now I need to find y. I know that x = -1. Substitute -1 for x into y = x +1. When I substitute -1 for x, I find y = 0.
Solution: (-1, 0)	My solution is the x and y values written as an ordered pair.
Step 4: Check -x +y = 1 -(-1) +0 = 1 1 =1 ☺	Substitute the values into each equation and check! 2x +y = -2 2(-1) +0 =-2 -2 = -2 ☺

Graphing Method:



Correlation:

Positive Correlation-If the data points make a straight line going from the origin out to high x- and y-values, then the variables are said to have a positive correlation.

Negative Correlation-If the line goes from a high-value on the y-axis down to a high-value on the x-axis, the variables have a negative correlation.

No Correlation-If the data is all over the graph with no pattern then the variables have no relationship and thus, no correlation.

Correlation Coefficient- used to indicate the relationship of two random variables. It provides a measure of the strength and direction of the correlation varying from -1 to +1.

Positive values indicate that the two variables are positively correlated, meaning the two variables vary in the same direction.

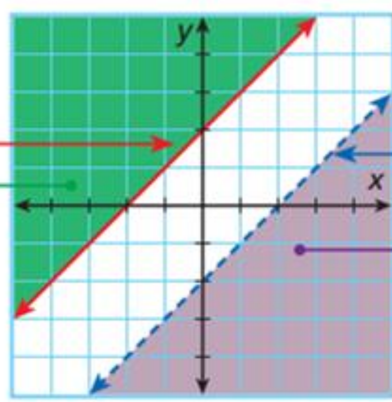
Negative values indicate that the two variables are negatively correlated, meaning the two variables vary in the contrary direction.

Values close to +1 or -1 reveal the two variables are highly related.

Graphing Linear Inequalities

When the inequality is written as $y \leq$ or $y \geq$, the points on the boundary line are solutions of the inequality, and the line is **solid**.

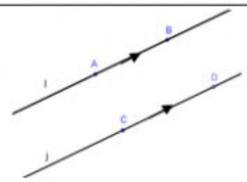
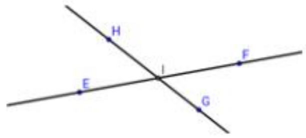
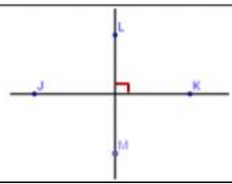
When the inequality is written as $y <$ or $y >$, the points on the boundary line are not solutions of the inequality, and the line is **dashed**.



When the inequality is written as $y >$ or $y \geq$, the points **above** the boundary line are solutions of the inequality.

When the inequality is written as $y <$ or $y \leq$, the points **below** the boundary line are solutions of the inequality.

Parallel & Perpendicular Lines

Characteristics	What You Draw
Parallel lines never cross and stay the same distance apart. They are coplanar. They have 0 points in common.	
Intersecting lines pass through the same point. They have one point in common.	
Perpendicular lines intersect at right angles. They have one point in common.	

<u>Parallel Lines</u>

-never intersect; **SAME SLOPE**

<u>Perpendicular Lines</u>

-intersect to form a right angle (90 degrees)

-The slopes of perpendicular lines are opposite reciprocals

(AKA- FLIP THE FRACTION AND FLIP THE SIGN)

Arithmetic and Geometric Sequences:

Arithmetic Sequence- goes from one term to the next by always adding (or subtracting) the same value.

Example 1: 2, 5, 8, 11, 14,... (add 3 each time)

Example 2: 7, 3, –1, –5,... (subtract 4 each time)

Geometric Sequence- goes from one term to the next by always multiplying (or dividing) by the same value.

Example 1: 1, 2, 4, 8, 16,... (multiply by 2 each time)

Example 2: 81, 27, 9, 3, 1, 1/3,... (divide by 3 each time)

Writing NEXT-NOW Equations

<u>Linear Example:</u>

y = $300x + $40,000 in NEXT-NOW form:

NEXT = NOW + 300, starting at $40,000

<u>Exponential Example:</u>

$y = 300(1.05)^x$ in NEXT-NOW form:

NEXT = NOW(1.05) Starting at 300

RECURSIVE AND EXPLICIT FORMULA

Let n= the term number in the sequence

Let A(n) = the value of the nth term of the sequence

Let d= the common difference

Recursive Formula

A(n) = A(n – 1) + d

Explicit Formula

A(n) = A(1) + (n – 1)d

LINEAR, EXPONENTIAL AND QUADRATIC TABLES:

When given a table you should find the difference for the x and y values.
What's happening to the x each time? What's happening to the y each time?

Linear Example:

(same difference on x and y every time; ratio or slope is constant)

x	*y*
0	5
1	1
2	-3
3	-7
4	-11

difference of *y* - values

- 1 - 5 = **-4**
- -3 - 1 = **-4**
- -7 +3 = **-4**
- -11+7 = **-4**

Exponential Example:

You are either multiplying be the same number each time or dividing by the same number (multiplying by a fraction)

In the example below they are x3 each time

x	*y*
0	4
1	12
2	36
3	108
4	324

ratio of *y* - values

- $\frac{12}{4}$ = **3**
- $\frac{36}{12}$ = **3**
- $\frac{108}{36}$ = **3**
- $\frac{324}{108}$ = **3**

Quadratic Example:

(Double Difference)

difference of *x*– values	*x*	*y*	difference of *y*– values
1 - 0 = **1**	0	5	-1 + 4 = **3**
3 - 1 = **2**	1	10	2 +1 = **3**
4 - 3 = **1**	3	20	5 – 2 = **3**
6 - 4 = **2**	4	25	8 – 5 = **3**
	6	35	

x	$y = x^2$
0	0
1	1
2	4
3	9
4	16
5	25
6	36

difference of *y*– values

- 1 – 0 = **1**
- 4 – 1 = **3**
- 9 – 4 = **5**
- 16 – 9 = **7**
- 25 – 16 = **9**
- 36 – 25 = **11**

difference of differences

- 3 – 1 = **2**
- 5 – 3 = **2**
- 7 – 5 = **2**
- 9 – 7 = **2**
- 11 – 9 = **2**